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Introduction to "A Numerical Method for the Study of the Circulation of the World Ocean"

Bryan's numerical formulation of a world ocean model [3] has endured long and well, not only because it was the first of its kind but also because it was suited to meet large computational challenges stretching three or more decades into the future. This short introductory article summarizes the history of the model's use, the model's features that were noteworthy and have stood the test of time, and the features that have been refined or replaced in later formulations. Perspectives on ocean modeling can be found in the review articles by Semtner [34] and McWilliams [27].

A BRIEF HISTORY OF THE MODEL USE

Kirk Bryan, together with the late Michael Cox, developed a versatile ocean circulation model during the 1960s at the NOAA Geophysical Fluid Dynamics Laboratory (GFDL). In 1969, Bryan published the formulation in the *Journal of Computational Physics;* this was the very first ocean model to permit general coastlines, arbitrary bathymetry, and multiple connectivity, as well as having full nonlinearity with temperature and salinity included. The equations were simplified through hydrostatic, Boussinesq, and incompressibility assumptions, and external gravity waves were removed by imposing a rigid lid. The second-order finite differencing of the 3-d governing equations in spherical coordinates was designed to be energy and property conserving. Ocean depth at each location was treated by including grid boxes only down to that depth. Each time step, a 2-d ancillary elliptic problem related to the rigid lid was solved by iteration. This established a paradigm that still serves ocean modeling and related fields such as climate modeling nearly 30 years later. The model is often referred to as the GFDL model.

It became clear in the early 1970s that the computational demands of physical ocean models are enormous and that major compromises in terms of grid size and time integration would have to be made. Cox's study [10] of the World Ocean with a 2° latitude–longitude grid and 9 levels could only be run for a few years over many months of elapsed time. As a result, rather coarse grids became the norm when climate time scales were involved, and grid sizes of 3°–5° were common into the late 1980s. Nevertheless, many interesting ocean problems were attacked using the Bryan formulation over a 20-year period, even including a few with grid sizes approaching 1° (for examples, see [5, 9, 12, 28, 32, 41]). Overall, the numbers of published studies carried out with the model grew from about 3 per year early in the period to 12 per year by the late 1980s.

Much of the energy of the ocean is concentrated at a small physical length scale known as the radius of deformation, which varies from about 1° near the equator to 1/10° or even 1/20° at high latitudes. This is the scale of intense boundary currents as well as transient eddies, and these phenomena are of considerable importance to the larger-scale dynamics. Early studies using grid sizes near the radius of deformation had to be carried out mainly with other models based on filtered equations and adiabatic dynamics in small idealized domains. However, advances in computing power allowed some degree of eddy representation in basin-wide or even global calculations using Bryan's formulation during the late 1980s. Studies with 1/4° to 1/2° grids were carried out for the Southern Ocean, [19], the North Atlantic ("CME" of Bryan and Holland, [6]), and the World Ocean ("POCM" of Semtner and Chervin [36, 37]. Even though these integrations could usually only span a few decades after starting from observed density fields, the opportunity to make close comparisons with satellite and *in situ* measurements—and then to find considerable agreement—generated new interest in the Bryan formulation.

During the 1990s, interest in the Bryan formulation has expanded in response to greater emphasis on ocean problems and the availability of increasingly powerful computers. This has happened even as other formulations, based mainly on different geometric representations (e.g., models with coast- or terrain-following coordinates or density coordinates), have matured and prospered. As a result, the number of publications citing [3] has increased dramatically, starting from 24 in 1991 and rising to 67 in 1995. At present, three major government laboratories use derivatives of Bryan's model for both global-ocean and climate studies; those are the GFDL as mentioned above, the National Center for Atmospheric Research (NCAR), and Los Alamos National Laboratory (LANL). Numerous university groups and many international researchers also use the formulation, sometimes relying on computer programs developed and freely distributed by the larger laboratories. In fact, GFDL maintains the Modular Ocean Model (MOM), while LANL has developed the Parallel Ocean

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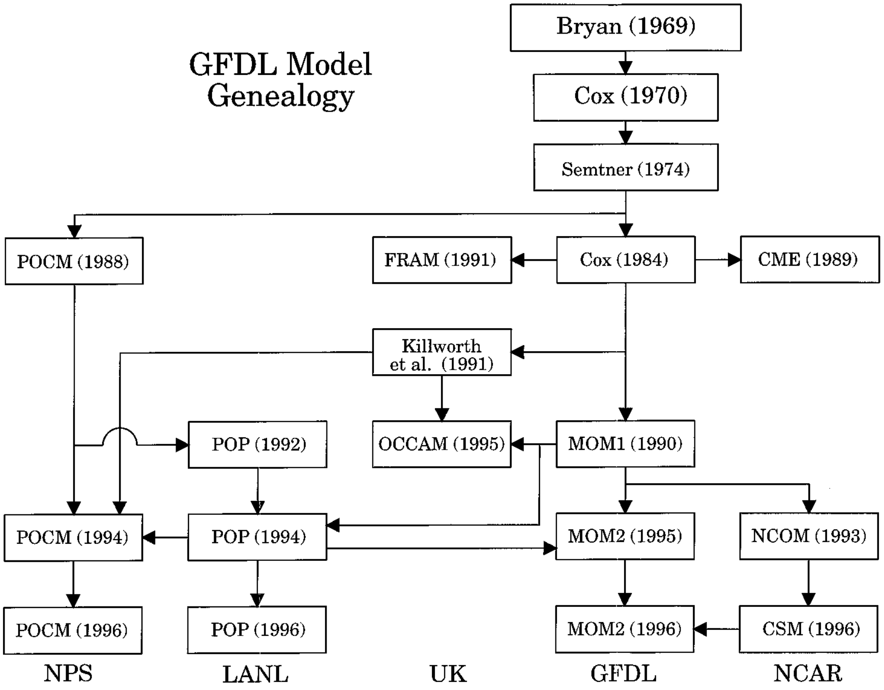

FIG. 1. Genealogy of the Bryan (1969) ocean model developed at the Geophysical Fluid Dynamics Laboratory (GFDL). Some of the boxes refer to published papers listed in the references. Other boxes refer to model versions by acronyms defined in the text and to specific years in which they reached new levels of maturity.

Program (POP), and NCAR has built upon MOM to provide its own ocean model (NCOM) within a larger Climate System Model (CSM). At the university level, there is the Parallel Ocean Climate Model (POCM) at the Naval Postgraduate School, and on an international level, there is the Ocean Circulation and Climate Advanced Model (OCCAM) in the United Kingdom, as well as major projects in Germany and Australia.

A schematic of modeling activities is shown in Fig. 1, which depicts the evolution of the Bryan model [3] in various forms up to the present day. Vertical lines show development primarily within the same institution. Horizontal lines show paths of external communication, whereby modeling expertise and improvements in the basic formulation have been extensively shared among groups. As a result, the efforts remain methodologically similar to one another even though they have evolved along separate paths. These paths are often defined by the nature of the physical problems being addressed or by the architecture of computers being used in the individual institutions. Roughly speaking, the left side of the diagram is oriented more toward high-resolution studies of multidecadal extent in a parallel-computing environment, while the right side of the diagram is oriented more toward lower-resolution studies on century or longer time scales with less emphasis on parallelism. Evidently, the passage of nearly 30 years has only enhanced the range of applicability for the basic method on which the models are founded.

WHY WAS THE FORMULATION A SUCCESS?

The 1969 paper was an immediate success because it filled a need for a nonlinear numerical model that could simulate aspects of real ocean circulation. The abilities to represent the irregular geometry of ocean coasts and depths and to have density and pressure as a function of predicted temperature and salinity were major improvements that would not be duplicated in other model formulations for at least a decade.

The paper was written in a didactic fashion, with clear explanations and examples to illustrate concepts unfamiliar to many oceanographers at the time. The model has energetic consistency and other desirable conservation properties. The 1969 paper is so clearly written that it can be used as a recipe for a computer program, which can include the energetic balances as a cross-check for programing errors. Following Cox's first program for the model, another emerged that was capable of running efficiently on the new breed of vector computers; it was developed at and then rather widely circulated from UCLA by Semtner [33]. Further versions, emanating from GFDL, featured more options and regular updates to users. These began with the Cox model [11] and continued with MOM1 of Pacanowski *et al.* [29] and later MOM2 [30]. Adaptations to parallel-vector and massively parallel computers were made with POCM by Chervin and Semtner [8], with POP by Smith *et al.* [38], and with OCCAM by Webb *et al.* [44]. It has become quite convenient for users to adopt a version of the model and then spend the majority of their time on oceanic issues rather than on algorithmic or programming issues. This has built broad support from the community.

Of course, the initial satisfaction with Bryan's formulation would have dissipated if significant defects were found over time. However, most of the purely hydrodynamic and numerical aspects of the formulation have withstood the test of time. Second-order time and space differencing continue to be serviceable choices. Time-truncation errors are small for the low-frequency phenomena of most interest, and high-order space differencing would cause difficulties in setting boundary conditions at walls. Of some importance, LANL investigators found that a standard meteorological method of ''pressure averaging'' allowed doubling the time step (R. Smith, personal communication, 1994). One concern arose earlier on the arrangement of grid points, which was thought to require modification for high-resolution studies. However, vertical noise uncovered with the alternative scheme (M. Cox, personal communication, 1979) and examination of numerical dispersive effects of planetary waves [15] showed the wisdom of the initial gridding at both high and low resolution. In the opinion of the present author, the ability to represent a broad class of oceanic phenomena has not been significantly surpassed by newer hydrodynamic formulations of other investigations, even though each type of model can be shown to have some specific strengths and weaknesses. In actuality, many of the deficiencies in the Bryan formulation have been ameliorated by higher resolution and/or improved process parameterizations.

WHAT PARTS OF THE MODEL HAVE CHANGED?

Probably the most significant algorithmic change has to do with the vertically integrated flow and the associated 2-d problem to be solved every time step. The rigid-lid method and its solver were ill-behaved in the presence of rugged topography; successful alternative methods to deal directly with a free surface were provided by Killworth *et al.* [23] using subcycling of small time increments and by Dukowicz and Smith [17] using an implicit method. As a result, the model can easily handle rugged topography without presmoothing, and it also no longer needs a special treatment of every island. The simulated elevation of the free surface can be directly compared with satellite altimeter measurements; and considerable agreement has been found [20, 40]. The satellite measurements include the seasonal cycle of thermal expansion in the upper ocean, and although this effect is not specifically allowed in the volume-preserving model, the effect has been shown to be negligible in most circumstances [16].

Further improvements are available that deal with specific inaccuracies. Early on, the empirical equation of state was found to be better approximated by quadratic polynomials at each level than by a single rational formula [4]. An improved method for computing vertical velocity in columns of momentum variables has been developed by Webb [43]. Methods of treating gradual changes in ocean depth through a variable-thickness bottom grid box have been developed by various investigators, e.g., Semtner and Mintz [35]. The north polar singularity, which has often been treated by longitudinal filtering of tendencies in variables near the pole, can be avoided either by transforming the pole into North America [39] or by using rotated spherical coordinates for the Arctic and North Atlantic [18]. Computational dispersion related to centered advection can be ameliorated by using third-order differencing [22]. Finally, difficulties in integrating temperature and salinity to long-term equilibrium can be overcome by acceleration techniques in coarse-grid models, even when a seasonal cycle is involved [14].

Changes in process parameterizations are helping to improve the realism of model simulations. At high resolution, the use of biharmonic operators in place of Laplacian ones for horizontal viscous and diffusive effects was implemented by Semtner and Mintz [35]. This allows eddies to form spontaneously and to carry out the mixing of properties in a dynamically more correct manner than before. At coarse resolution, a physical parameterization for the effects of unresolved eddies has now been developed by Gent and McWilliams [21]. Their parameterization includes isopycnal mixing of properties in a fashion similar to an earlier method of Redi [31], but the former also includes eddy-induced advection of tracers and additional diapycnal mixing to account more fully for the missing eddies [13]. The introduction of a hybrid vertical coordinate that approximately follows isopycnal surfaces may facilitate the isopycnal mixing of properties in future versions of the model (J. Dukowicz, personal communication, 1997). Finally, new treatments of near-surface boundary layers are leading to better representations of vertical exchanges than was possible in earlier versions of the model [24].

SUMMARY

As has been described above, the model of Bryan [3] continues to be used in modified forms at many laboratories, universities, and diverse international sites. Nearly 30 years after the model's inception, new successes are being demonstrated with fine grids of about 1/6° for the Atlantic Ocean [1, 7] and the Global Ocean [25]. An ongoing LANL simulation of North Atlantic boundary current separation and mid-ocean eddy dynamics shows significant improvement in representing these processes with a grid mesh finer than 1/10°, as well as a very close agreement with satellite altimeter measurements (R. Smith, M. Maltrud, and F. Bryan, personal communication, 1997). Even at rather coarse resolution, the ocean model formulation plays a crucial role in GFDL climate studies [26], and it promises to be an effective and progressively more quantitative tool in NCAR climate studies at improved resolution [2, 42]. Thus, it remains one of the best all-around ocean models after all these years.

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Albert J. Semtner

Naval Postgraduate School